RAID : Redundant Array of Inexpensive Disks.

Storage-efficient Redundancy.

RAID-0. → Striping.

Chunk ← 0 → Stripe.

D_0	D_1	D_2	D_3
$\boxed{0}$	1	2	3
4	5	6	7
8	9	10	11
⋮	⋮	⋮	⋮

How many disks can fail?

0

parallel I/O.

RAID-1 (mirroring).

D_0	D_1	D_2	D_3
0	0	1	1
2	2	3	3
4	4	5	5
⋮	⋮	⋮	⋮

How many disks can fail?

Best case : 2.

Worst case : 1.

RAID-4 (Parity). Parity Disk.

	D_0	D_1	D_2	D_3	D_4.
Stripe.	0	1	2	3	P_0
	4	5	6	7	P_1
	8	9	10	11	P_2.

Example based on +:

$\cancel{D_0}$	D_1	D_2	D_3	P

$4 + 3 + 0 + 2 \quad ? = 9$

$9 - 3 - 0 - 2 = 4$

XOR (exclusive OR).

	b_0	b_1	b_2	b_4	
XOR (	0	0	1	1)	= 0
XOR (	0	1	0	0)	= 1

Num. 1s even = 0

Num 1s odd. = 1

OR.

0 OR 1 = 1

AND

0 AND 1 = 0

XOR.

0 XOR 1 =

$0 \oplus 1 = 1$

Encoding.

	$\cancel{d_0}$	d_1	d_2	d_3	$\boxed{P}$
XOR (	$\cancel{00}$	10	11	10)	= 11
XOR (	$\cancel{10}$	01	00	10)	= 01

Decoding $10 \oplus 11 \oplus 10 \oplus 11 = \boxed{00}$

$01 \oplus 00 \oplus 10 \oplus 01 = \boxed{10}$

Can tolerate 1 disk failure in this case!

RAID-6. (Reed-Solomon Coding).

6 Data Disks
2 Parity Disks.

d_0	d_1	d_2	d_3	d_4	d_5	p	q
0	1	2	3	4	5	p_0	q_0
6	7	8	9	10	11	p_1	q_1

parity check matrix $\underline{(8\times 6)} \cdot (6\times 1) = (8\times 1)$.

$$\begin{array}{l} \text{D.M.} \\ \\ \\ \\ \\ \\ \text{PCM.} \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 32 & 16 & 8 & 4 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ p \\ q \end{bmatrix}$$

code word → Encoding

$p = \boxed{d_0 \oplus d_1 \oplus d_2 \oplus d_3 \oplus d_4 \oplus d_5}$

$q = \boxed{32d_0 \oplus 16d_1 \oplus 8d_2 \oplus 4d_3 \oplus 2d_4 \oplus d_5}$

Parity Check Matrix

$$
\left[\begin{array}{cccccc|cc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
32 & 16 & 8 & 4 & 2 & 1 & 0 & 1
\end{array}\right]
\times
\begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ p \\ q \end{bmatrix}
$$

$$d_0 \oplus d_1 \oplus d_2 \oplus d_3 \oplus d_4 \oplus d_5 \oplus p = 0$$

$$32 d_0 \oplus 16 d_1 \oplus 8 d_2 \oplus 4 d_3 \oplus 2 d_4 \oplus d_5 \oplus q = 0$$

Decoding

$$S_0 = d_0 \oplus d_2 \oplus d_3 \oplus d_5 \oplus p = d_1 \oplus d_4$$

$$S_1 = 32 d_0 \oplus 8 d_2 \oplus 4 d_3 \oplus d_5 \oplus q = 16 d_1 \oplus 2 d_4$$

$$S_0 = d_1 \oplus d_4$$

$$S_1 = 16 d_1 \oplus 2 d_4$$

polynomial solver

Gaussian Elimination

Matrix Inversion