

RAID : Redundant Array of Inexpensive Disks.

Storage-efficient Redundancy

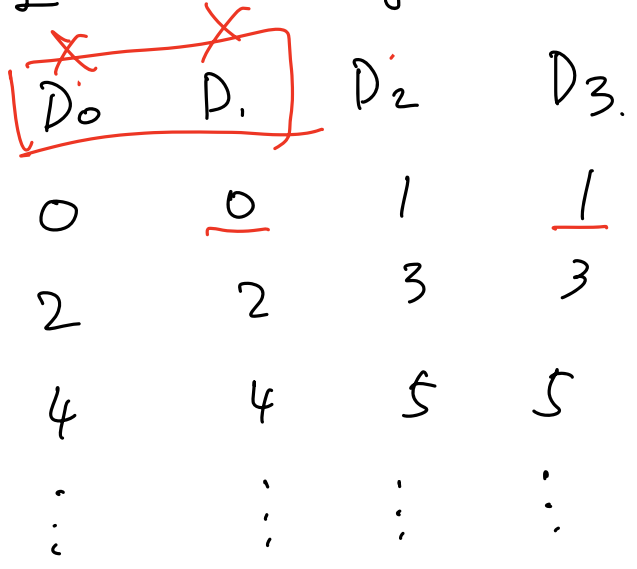
RAID-0. → Striping.



How many disks can fail?

0  
parallel I/O.

RAID-1 (mirroring).



How many disks can fail?

Best case : 2.

Worst case : 1.

# RA2D-4 (Parity). Parity Disk.

	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
Stripe.	0	1	2	3	P <sub>0</sub>
	4	5	6	7	P <sub>1</sub>
	8	9	10	11	P <sub>2</sub>

Example based on +:

$$\begin{array}{cccccc}
 \text{X} & & & & & \\
 \text{D}_0 & \text{D}_1 & \text{D}_2 & \text{D}_3 & \text{P} & \\
 \text{4} & + 3 & + 0 & + 2 & ? & = 9
 \end{array}$$

$$9 - 3 - 0 - 2 = \text{4}$$

XOR (exclusive OR):

$$\text{XOR}(b_0 \ 0 \ b_1 \ 0 \ b_2 \ 1 \ b_3 \ 1) = 0$$

$$\text{XOR}(0 \ 1 \ 0 \ 0) = 1$$

Num. 1s even = 0

Num 1s odd. = 1

OR.

$$0 \text{ OR } 1 = 1$$

AND

$$0 \text{ AND } 1 = 0$$

XOR.

$$0 \text{ XOR } 1 =$$

$$0 \oplus 1 = 1$$

Encoding.

$$\begin{array}{cccccc}
 \text{X} & & & & & \\
 \text{d}_0 & \text{d}_1 & \text{d}_2 & \text{d}_3 & \boxed{\text{P}} & \\
 \text{XOR}(\text{00} & 10 & 11 & 10) = 11 \\
 \text{XOR}(\text{10} & 01 & 00 & 10) = 01
 \end{array}$$

Decoding

$$\begin{matrix} 10 \oplus 11 \oplus 10 \oplus 11 = 00 \\ 01 \oplus 00 \oplus 10 \oplus 01 = 10 \end{matrix}$$

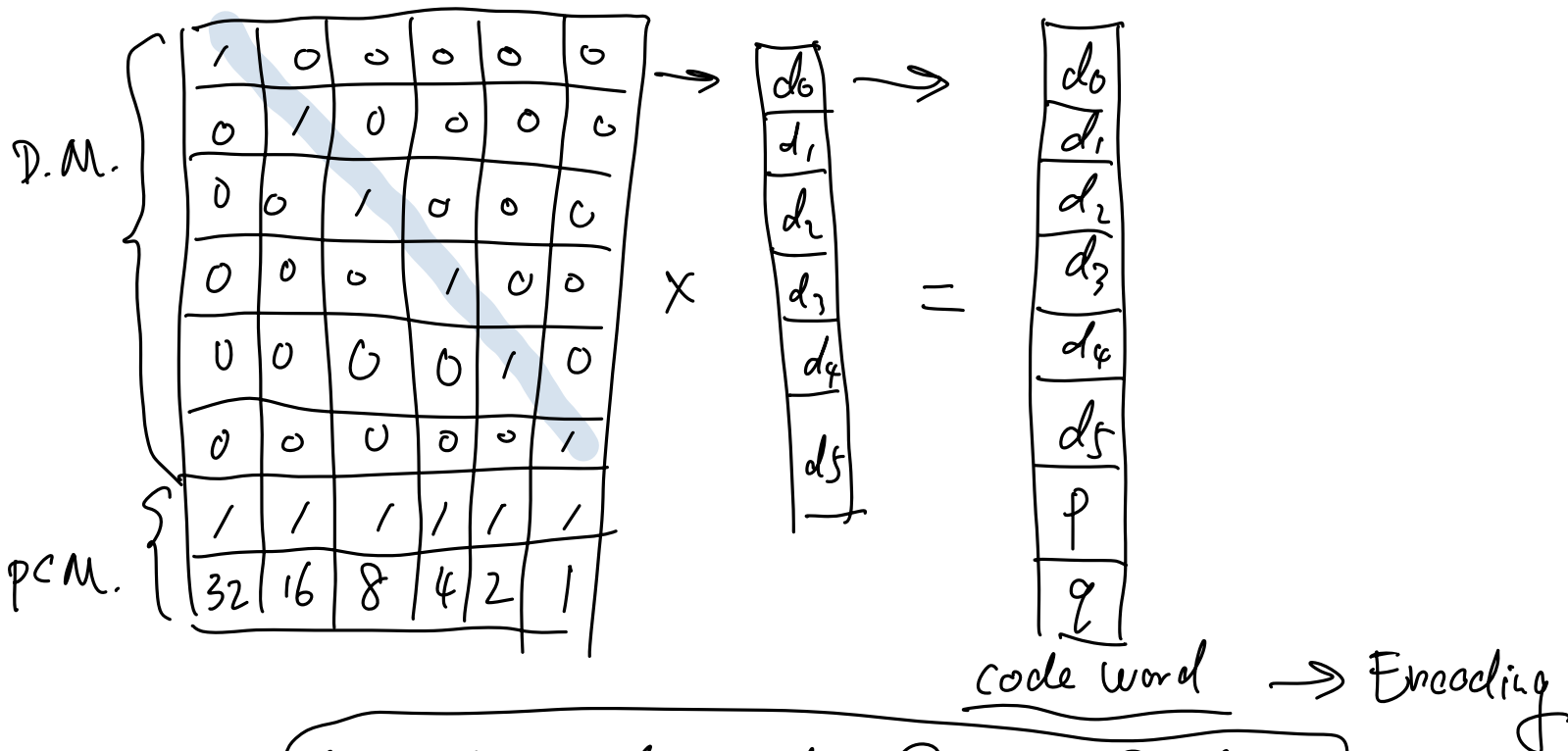
(can tolerate 1 disk failure in this case!)

RA2D-6. (Reed-Solomon coding).

6 Data Disks  
2 Parity Disks.

	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	P	Q
	0	1	2	3	4	5	$P_0$	$Q_0$
	6	7	8	9	10	11	$P_1$	$Q_1$

parity check matrix  $(8 \times 6) \cdot (6 \times 1) = (8 \times 1)$ .



$$P = d_0 \oplus d_1 \oplus d_2 \oplus d_3 \oplus d_4 \oplus d_5$$

$$Q = 32d_0 \oplus 16d_1 \oplus 8d_2 \oplus 4d_3 \oplus 2d_4 \oplus d_5$$

# Parity Check Matrix

1	1	1	1	1	1	1	0
32	16	8	4	2	1	0	1

↑      ↑

X

$d_0$
$d_1$
$d_2$
$d_3$
$d_4$
$d_5$
$p$
$q$

X

$$d_0 \oplus d_1 \oplus d_2 \oplus d_3 \oplus d_4 \oplus d_5 \oplus p = 0$$

$$32d_0 \oplus 16d_1 \oplus 8d_2 \oplus 4d_3 \oplus 2d_4 \oplus d_5 \oplus q = 0$$

Decoding

$$S_0 = d_0 \oplus d_2 \oplus d_3 \oplus d_5 \oplus p = d_1 \oplus d_4$$

$$S_1 = 32d_0 \oplus 8d_2 \oplus 4d_3 \oplus d_5 \oplus q = 16d_1 \oplus 2d_4$$

$$S_0 = \underbrace{d_1}_{\text{circled}} \oplus \underbrace{d_4}_{\text{circled}}$$

$$S_1 = 16d_1 \oplus 2d_4$$

polynomial solution

Gaussian Elimination

Matrix Inversion.