Memory Management: Page Replacement Policies: Belady's Optimal

CS 571: Operating Systems (Spring 2020)

Lecture 8c

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Some material taken/derived from:

- Wisconsin CS-537 materials created by Remzi Arpaci-Dusseau.

Belady's Optimal

OPT: The Optimal Replacement Policy

- Many years ago Belady demonstrated that there is a simple policy (OPT or MIN) which always leads to fewest number of misses
- Idea: evict the page that will be accessed furthest in the future
- Assumption: we know about the future
- Impossible to implement OPT in practice!

- But it is extremely useful as a practical best-case baseline for comparison purpose

Proof of Optimality for Belady's Optimal Replacement Policy

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.307.7603&rep=rep1&type=pdf

A Short Proof of Optimality for the **MIN** Cache Replacement Algorithm

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Abstract

The **MIN** algorithm is an offline strategy for deciding which item to replace when writing a new item to a cache. Its optimality was first established by Mattson, Gecsei, Slutz, and Traiger [2] through a lengthy analysis. We provide a short and elementary proof based on a dynamic programming argument.

Keywords: analysis of algorithms, on-line algorithms, caching, paging

1 The MIN Algorithm

OPT the Optimal

- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

OPT the Optimal

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- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0			
1			
2			
0			
1			
3			
0			
3			
1			
2			
1			

assume cache size 3

OPT the Optimal

- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0			
1			
3			
0			
3			
1			
2			
1			

assume cache size 3

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- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3			
0			
3			
1			
2			
1			

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- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3			
0			
3			
1			
2			
1			

assume cache size 3

What to evict??

OPT the Optimal

- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3			
0			
3			
1			
2			
1			

assume cache size 3

What to evict??

Page 2 happens to be the one that will be accessed furthest in future!

OPT the Optimal

- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3	Miss	2	0, 1, 3
0			
3			
1			
2			
1			

assume cache size 3

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- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3	Miss	2	0, 1, 3
0	Hit		0, 1, 3
3	Hit		0, 1, 3
1	Hit		0, 1, 3
2			
1			

assume cache size 3

OPT the Optimal

- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3	Miss	2	0, 1, 3
0	Hit		0, 1, 3
3	Hit		0, 1, 3
1	Hit		0, 1, 3
2			
1			

assume cache size 3

What to evict??

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- Idea: evict the page that will be accessed furthest in the future
- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3	Miss	2	0, 1, 3
0	Hit		0, 1, 3
3	Hit		0, 1, 3
1	Hit		0, 1, 3
2			
1			

assume cache size 3

Page 1 will be accessed right after page 2. Hence 1 is safe!

What to evict??

OPT the Optimal

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- Example workload: 0 1 2 0 1 3 0 3 1 2 1

Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3	Miss	2	0, 1, 3
0	Hit		0, 1, 3
3	Hit		0, 1, 3
1	Hit		0, 1, 3
2	Miss	3	0, 1, 2
1			

assume cache size 3

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- Idea: evict the page that will be accessed furthest in the future
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Access	Hit/Miss?	Evict	Resulting Cache State
0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3	Miss	2	0, 1, 3
0	Hit		0, 1, 3
3	Hit		0, 1, 3
1	Hit		0, 1, 3
2	Miss	3	0, 1, 2
1	Hit		0, 1, 2

assume cache size 3

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0	Miss		0
1	Miss		0, 1
2	Miss		0, 1, 2
0	Hit		0, 1, 2
1	Hit		0, 1, 2
3	Miss	2	0, 1, 3
0	Hit		0, 1, 3
3	Hit		0, 1, 3
1	Hit		0, 1, 3
2	Miss	3	0, 1, 2
1	Hit		0, 1, 2

assume cache size 3

The optimal number of cache hits is **6** for this workload!